# FRACTAL DIMENSION OF ELECTRON

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Abstract: *We would like to find a better and plausible structure of the electron as a torus and fractal structure. However it is in contradiction with generally accepted knowledge, where the electron has not a structure. Actual properties of the electron cannot be explained by point-like models. This paper is an attempt to improve our previous calculations of the radius of the electron and its fractal dimension. The vortex-fractal theory could possibly help with explanation what the charge, the electron, the proton, the electromagnetic field, etc. actually are.*

Keywords: *vortex-fractal-ring physics, structure of electron, charge, size of electron, fractal dimension, spin, torus*

## 1 Introduction

The Standard Model of Elementary Particles treats the electron as a quantum object with wave-particle duality and inherent properties arbitrarily assigned in order to correspond with various empirical data. These properties are not related to any physical structure or internal motions of the electron itself. For many physicists, an electron is like a mysterious ‘black box’ that no one can enter in order to discover why the electron produces certain line spectra, spin, mass, a magnetic moment, wavelength, etc.

Classical approach to particle physics is based on three ideas: the idea that elementary particles are physical objects with structure, and the idea that these physical objects are fundamentally electrical in character. Furthermore, the discovery that an electron has magnetic properties suggests to classical physicists that the electric charge carried by an electron must be in motion [1].

The first spinning charged ring (SCR) model was proposed for the electron by A. L. Parson in 1915 [1]. His model consisted of charge moving along the circumference of a thin torus ring. While the preceding model (a sphere) had only one degree of freedom, radius R, Parson’s spinning ring has three degrees of freedom, radius R, half-thickness r, and rotation rate ω, providing more opportunity for characteristics of the ring model to conform to the measured parameters of the electron. Nevertheless, Parson’s ring electron and more recent SCR models have one common defect: Charge cannot be confined to a surface sheet of zero volume, because compression of each segment of charge repels all other segments with infinite force in accordance with Coulomb’s Law, and in this case the electron would explode.

## 2 Finite size of electrons

The charge cannot be confined to a point, line, or plane of zero volume, because compression of each segment of charge repels all other segments in accordance with Coulomb’s Law. Thus the electron is elastic, finite in size, and finite in mass-energy. Ivan Sellin wrote in 1982 that “A good theory of electron structure still is lacking. There is still no generally accepted explanation for: why electrons do not explode under the tremendous Coulomb repulsion forces in an object of small size?” [1].

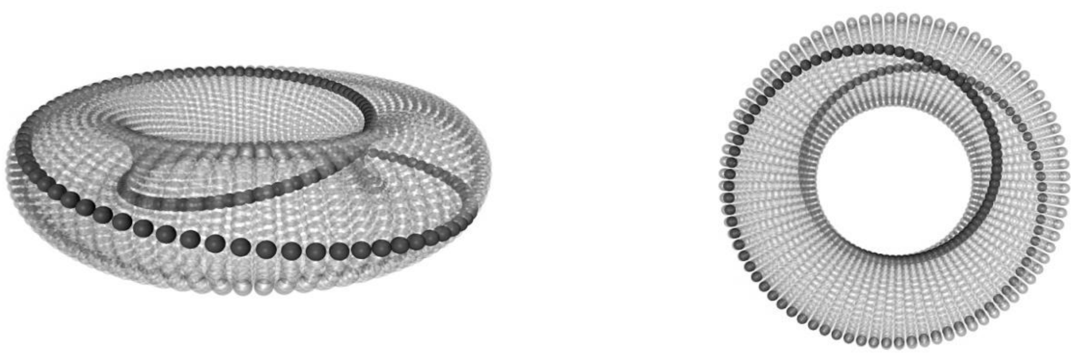

Fig. 1 The torus structure of the electron with spin ½

### 2.1 Radius of the electron

The black spheres on Fig.1 represent the motion of sub-electron rings moving helically on the surface of the torus. There is $N_1/2$ double loops and every double loop has $2N_2$ sub-electrons ${}^{-1}e$. Number of sub-electrons is $N=N_1N_2=42x42=1764$ ($N_1=N_2$) [3]. Derivation of the size of the electron with torus structure (see Fig.1) can be done by geometry on Fig.2:

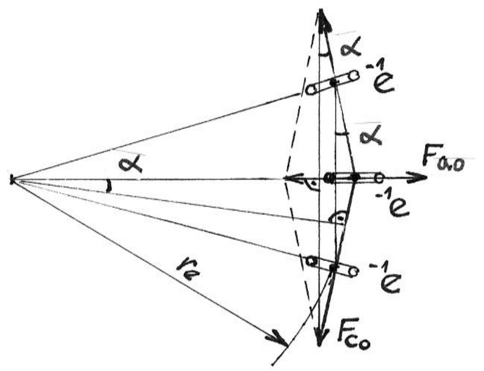


Fig. 2 Forces between two sub-electrons ${}^{-1}$e inside the electron ${}^{0}$e:

$$\sin\alpha = \frac{\frac{F_{a0}}{2}}{F_{C0}} = \frac{F_{a0}}{2F_{C0}} \tag{1}$$

where $F_{a0}$ is the acceleration force, $F_{c0}$ is the Coulomb's force, ${}^{-1}$e is the first ring substructure of the electron.

$$\sin\alpha = \frac{\frac{2\pi r_e}{N_2}\frac{1}{2}}{r_e} = \frac{\pi}{N_2} \tag{2}$$

where $r_e$ is the radius of the electron, N is number of fist ring substructures.

$$\sin\alpha = \frac{F_{a0}}{2F_{C0}} = \frac{\pi}{N_2} \tag{3}$$

$$F_{a0} = \frac{2\pi}{N_2} F_{a0} \tag{4}$$

$$\frac{m_e}{N}\frac{v_e^2}{r_e} = \frac{m_e}{N_1N_2}\frac{v_e^2}{r_e} = \frac{2\pi}{N_2}\frac{\left(\frac{e}{S}\right)^2}{4\pi^2\varepsilon_0\left(\frac{2\pi r_e}{N_2}\right)^2} \tag{5}$$

where $m_e$ is mass of the electron, $1/S$ is scaling factor, $m_e/N$ is mass of the first ring substructure ${}^{-1}e$, $v_e$ is velocity of the subelectron ${}^{-1}e$. We suppose that there are $N$ substructures ${}^{-1}e$ inside the electron. Solving (5) for $r_e$:

$$r_e = \frac{e^2}{8\pi^2\varepsilon_0 m_e}\frac{1}{{v_e}^2}\frac{N_1N_2^2}{S^2} \tag{6}$$

To receive result that is not in contradiction with general accepted knowledge we use:

$$\pi = \frac{N_1N_2^2}{S^2} \qquad\qquad S = N_2\sqrt{\frac{N_1}{\pi}} \tag{7}$$

We receive the plausible radius $r_e$ of the electron. This formula is valid only for $v_e >> 0$:

$$r_e = \frac{e^2}{8\pi\varepsilon_0 m_e}\frac{1}{{v_e}^2} \tag{8}$$

The Lorentz factor is an expression which appears in several equations in special relativity. It arises from deriving the Lorentz transformations. Equation for mass of the electron in special relativity:

$$m_e = \frac{m_{e0}}{\sqrt{1-\frac{v_e^{\ 2}}{c^2}}} \tag{9}$$

where $m_{e0}$ is the mass of the electron for $v_e=0$

$$r_e = \frac{e^2}{8\pi\varepsilon_0 m_{e0}} \frac{1}{v_e^{\ 2}} \sqrt{1-\frac{v_e^2}{c^2}} \tag{10}$$

for $0< v_e << c$ in (22):

$$r_e = \frac{e^2}{8\pi\varepsilon_0 m_{e0}} \frac{1}{v_e^{\ 2}} \tag{11}$$

**2.2 Fractal dimension of the electron**

A fractal dimension is a ratio providing a statistical index of complexity comparing how detail in a pattern (strictly speaking, a fractal pattern) changes with the scale at which it is measured. Hausdorff "fractional" dimension $D$ is defined as:

$$D = -\frac{\ln N}{\ln \varepsilon} = \frac{\ln N}{\ln S} = \frac{\log N}{\log S} \tag{12}$$

where $N$ is number of substructures (number of new sticks), ε =1/S is scaling factor. For $S$ from (7) and $N_1 = N_2 = 42$ [3] is fractal dimension $D$:

$$S = \sqrt{\frac{N_2^3}{\pi}} = \sqrt{\frac{42^3}{\pi}} \approx 153.57 \qquad\qquad D = \frac{\log N}{\log S} \approx 1.48 \tag{13}$$

**2.3 Radius of the free electron**

To calculate the radius $r_e$ for $v_e=0$ it is necessary add a second part in (5). It means we must add a repulsion force in the levitation model (see Fig.2):

$$\frac{m_e}{N}\frac{v_e^2}{r_e} = \frac{2\pi}{N_2}\frac{\left(\frac{e}{S}\right)^2}{4\pi^2\varepsilon_0}\left[\frac{1}{\left(\frac{2\pi r_e}{N_2}\right)^2} - \frac{\left(\frac{d_{0ee}}{S}\right)^2}{\left(\frac{2\pi r_e}{N_2}\right)^4}\right] \tag{14}$$

Where $d_{0ee}$ is levitation distance between 2 sub-electrons:

$$d_{0ee}^{\ \ 2} = d_0^{\ 2}\frac{m_e}{m_p} \tag{15}$$

where $d_0$ is levitation distance between the proton and the electron for quantum number $n=1$, $m_e$ is mass of the electron, $m_p$ is mass of the proton. For the free electron the equation (14) with $v_e=0$ :

$$r_{e-\min} = d_0 \frac{1}{2\pi}\sqrt{\frac{\pi}{N_2}\frac{m_e}{m_p}} \approx \frac{d_0}{984} \tag{16}$$

**2.4 Size of the electron**

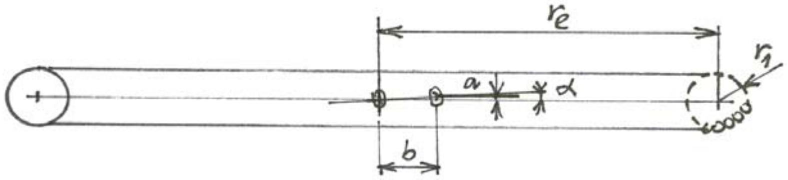


Fig. 3 Size of the electron

$$2\pi r_1 = N_1\frac{2r_e}{S} \qquad\qquad \frac{r_1}{r_e} = \frac{N_1 r_e}{\pi r_e S} = \frac{N_1}{\pi N_2\sqrt{\frac{N_1}{\pi}}} \approx \frac{1}{11.49} \tag{17}$$

$$tg\alpha = a\frac{1}{b} = 2\pi r_1 \frac{1}{2N_1}\frac{N_2}{2\pi r_e} \qquad \alpha \approx 0.343^o \tag{18}$$

## 3 Quantum model of the electron

On the circumference of the double loop with the radius $r_e$ (see Fig.1 and Fig.2) have to be $n$ of de Broglie's wavelengths $\lambda$ ($n$ is quantum number) which are created by sub-electrons with mass $m_e/N$:

$$2\cdot 2\pi r_e = 4\pi \frac{e^2}{8\pi\varepsilon_o \frac{m_e}{N}}\frac{1}{v_e^2} = n\lambda = n\frac{h}{\frac{m_e}{N}v_e} \tag{19}$$

$$\frac{e^2}{2\pi\varepsilon_o}\frac{1}{v_e} = nh \tag{20}$$

where $v_e$ is velocity of the sub-electron with mass $m_e/N$ and $v_e$ is at the same time the rotation velocity $v_{en}$ of the electron on level $n$:

$$v_{en} = \frac{1}{n}\frac{e^2}{2\varepsilon_o h} \tag{21}$$

For $n=1$ on the ground state the electron has maximal velocity $v_{e\text{-}max}$ :

$$v_{e-\max} = \frac{e^2}{2\varepsilon_o h} = \alpha c \cong 2180\, km/s \tag{22}$$

where $\alpha$ is the couple constant:

$$\alpha = \frac{e^2}{2\varepsilon_o hc} \tag{23}$$

$$\frac{c}{v_{e-\max}} = \frac{2\varepsilon_o hc}{e^2} = \frac{1}{\alpha} \approx 137.036 \tag{24}$$

Energy $E_{rn}$ of rotation of the electron on quantum level $n$:

$$E_{rn} = \frac{1}{2}\frac{m_e}{N}N\cdot v_{en}^2 = \frac{1}{n^2}\frac{m_e e^4}{8\varepsilon_o^2 h^2} \approx \frac{1}{n^2}13.6eV \tag{25}$$

For quantum number $n=1$ ionization energy $E_{io}$ :

$$E_{io} = \frac{m_e e^4}{8\varepsilon_o^2 h^2} \approx 13.6eV \tag{26}$$

Energy $E_n$ of levitation from [3] for levitation distance $d_{on}$ on level $n$: $\quad d = d_{on} = n^2 d_o \qquad (27)$

$$E_n = -\frac{e^2}{4\pi\varepsilon_o}\frac{1}{d}\left(1 - \frac{n^4 d_o^2}{3d^2}\right) = -\frac{e^2}{4\pi\varepsilon_o}\frac{1}{n^2 d_o}\frac{2}{3} = \frac{1}{n^2}18.13eV \qquad E_{i0} = \frac{3}{4}E_n \approx \frac{3}{4}18.13eV \approx 13.6eV \tag{28}$$

Energy $E_n$ of levitation is in balance with energy $E_{rn}$ of rotation. For the levitation distance $d_o$ in the hydrogen atom:

$$E_{rn} = -\frac{1}{n^2}13.6eV = -\frac{1}{n^2}\frac{m_e e^4}{8\varepsilon_o{}^2 h^2} = -\frac{e^2}{4\pi\varepsilon_o}\frac{1}{n^2 d_o}\frac{2}{3}\cdot\frac{3}{4} = -\frac{1}{n^2}\frac{m_e e^4}{8\varepsilon_o{}^2 h^2} \tag{29}$$

$$d_o = \frac{\varepsilon_o h^2}{\pi m_e e^2} = r_B \tag{30}$$

The Bohr radius $r_B$ has the same size as the distance $d_o \approx 5.29\cdot 10^{-11}m$ in our vortex-fractal-ring model [2], [5].
Since $n=1$ the radius $r_e$ of the electron is :

$$r_{e1} = \frac{e^2}{8\pi\varepsilon_0 m_e}\cdot\frac{1}{v_{e-\max}^2} = \frac{e^2}{8\pi\varepsilon_0 m_e}\cdot\frac{4\varepsilon_o{}^2 h^2}{e^4} = \frac{1}{2}\frac{\varepsilon_o h^2}{\pi m_e e^2} = \frac{1}{2}d_0 = 2.645\cdot 10^{-11}m \tag{31}$$

Since $1<n<10$ [for higher $n$ we must use more complex equation (14)]:

$$r_{en} = \frac{n^2}{2} d_0 = n^2 r_{e1} \tag{32}$$

## 4 The spin of the electron

It was discovered in 1925 that the electron has properties corresponding to its spin S. The spin of the electron is defined as angular momentum:

$$\vec{S} = m_e (\vec{r}_e \times \vec{v}_e) \tag{33}$$

For the spin on axis z:

$$S_z = \pm N \frac{m_e}{N} r_e v_e = \pm m_e r_e v_e \tag{34}$$

where $m_e$ is the mass of the electron, $r_e$ is the radius of the electron and $N$ is number of substructures inside the structure of the electron. For quantum number $n=1$:

$$r_{e1} = \frac{e^2}{8\pi\varepsilon_0 m_e} \cdot \frac{1}{v_{e-\max}^2} \tag{35}$$

$$v_{e-\max} = \frac{e^2}{2\varepsilon_0 h} \tag{36}$$

$$S_z = \pm m_e \frac{e^2}{8\pi\varepsilon_0 m_e} \cdot \frac{1}{v_{e-\max}^2} v_{e-\max} = \pm m_e \frac{e^2}{8\pi\varepsilon_0 m_e} \cdot \frac{1}{v_{e-\max}} = \pm m_e \cdot \frac{e^2}{8\pi\varepsilon_0 m_e} \cdot \frac{2\varepsilon_0 h}{e^2} = \pm \frac{1}{2} \cdot \frac{h}{2\pi} = \pm \frac{1}{2}\hbar = m_s \hbar \tag{37}$$

$$m_s = \pm \frac{1}{2} \tag{38}$$

The result in (37) is in coincidence with the generally equation for the spin, where $m_s$ is spin quantum number.

## 5 Magnetic field of permanent magnet

Some structures of atoms can be arranged by a suitable way that their magnetic field is oriented in one direction (e.i. as is shown on Fig.4)

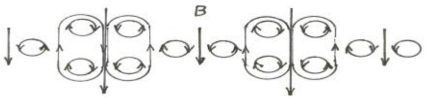


Fig.4 Explanation of the magnetic field of a permanent magnet

## 6 Examples of ring structures

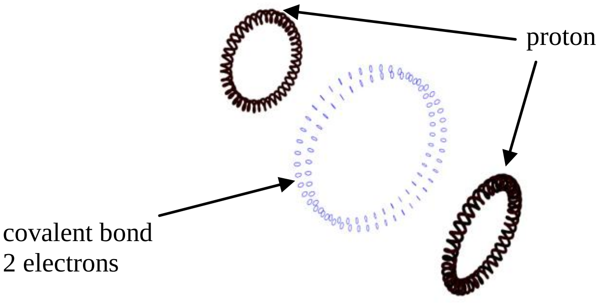


Fig. 5 Vortex-fractal ring structure of the hydrogen molecule

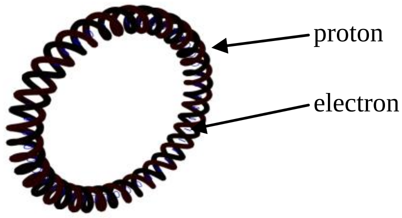


Fig. 6 Vortex-fractal ring structure of neutron (the electron is inside of the proton)

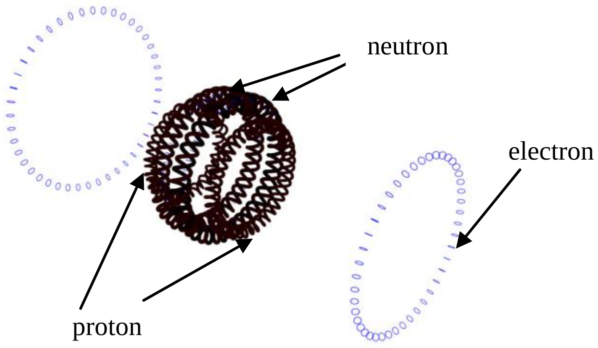


Fig.7 Vortex-fractal ring structure of the helium atom

Every proton has the double helix structure. It can explain its spin ½. Electric lines are perpendicular to magnetic lines. Electric and magnetic lines create a complex vortex structure which holds sub-parts of atoms (molecules) together (see Fig.5 and Fig.7).

## 7 Conclusion

The point-electron is still a dominant feature of the modern model of the electron. But the electron, proton, and neutron each have measured amounts of spin (angular momentum) and magnetic moment. These features can only exist because the particles have structure and a finite, non-zero size. Actual properties of the electron cannot be explain by point-like models used in theories such as relativity theory, quantum mechanics, and the Dirac theory of the atom. Comparison shows that only a physical model of the electron with finite size can explain the fundamental properties of the electron, i.e. charge, mass, spin, magnetic moment, and stability. Just as ordinary objects are composed of a material substance with size and shape, so also by the principle of unity and the philosophy of structuralism must an electron be composed of a material substance with a non-zero size and the specific structure. While points cannot provide a physical mechanism for the exchange of energy between particles, a finite-sized object will change size and shape in response to the presence of another object. Treating the electron as a torus structure rather than the point-like particle is a very challenging physical topic.

In future we will derive equations for other structures like is the proton and the neutron.

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